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**LETTER TO THE EDITOR**

# Co-operative two-channel Kondo effect

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**Abstract.** We discuss how the properties of a single-channel Kondo lattice model are modified by additional screening channels. Contrary to current wisdom, additional screening channels appear to constitute a relevant perturbation which destabilizes the Fermi liquid. When a heavy Fermi surface develops, it generates zero modes for Kondo singlets to fluctuate between screening channels of different symmetry, producing a divergent composite pair susceptibility. Additional screening channels couple to these divergent fluctuations, promoting an instability into a state with long-range composite order.

A puzzling question that arises in trying to understand heavy-fermion superconductors is how the localized moments seen in high temperature properties participate in the pair condensate [1]. In these systems a significant fraction of the entropy associated with the local moments appears to be involved with the superconducting condensation process: for $UBe_{13}$, the spin-condensation entropy is about $0.2k_B \ln 2$ per spin [2]. The concept of 'composite pairing', where a Cooper pair and local moment form a bound-state combination that collectively condenses, may provide a way of understanding this large spin-condensation entropy [3–5]. Recent studies of the one-dimensional Kondo lattice at strong coupling [6] and the infinite-dimensional two-channel Kondo lattice have both given indication of a composite pairing instability [7].

In this letter we discuss how the properties of a single-channel Kondo lattice model for heavy fermion systems are modified by coupling to additional screening channels. Current wisdom, based on the naive extrapolation from single impurity models [8, 9], regards these additional couplings to be irrelevant. We shall show that an entirely different state of affairs arises in a two-channel Kondo lattice where the scattering channels of different local symmetry are obliged to share a single Fermi sea. This allows for the possibility of *constructive* interference between the two channels which drives the development of composite order.

Consider a sea of conduction electrons coupled to an $N$-site lattice of spin-$\frac{1}{2}$ local moments via two channels:

$$H = H_0 + \sum_j \left\{ J_1 \psi^\dagger_{1j} \boldsymbol{\sigma} \psi_{1j} + J_2 \psi^\dagger_{2j} \boldsymbol{\sigma} \psi_{2j} \right\} \cdot \boldsymbol{S_j} \tag{1}$$

where $H_0 = \sum \epsilon_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma}$ describes a single electron band and $\psi^\dagger_{\Gamma j} = (\psi^\dagger_{\Gamma j\uparrow},\ \psi^\dagger_{\Gamma j\downarrow})$ is a two-component spinor

$$\psi^\dagger_{\Gamma j} = N^{-\frac{1}{2}} \sum_{\boldsymbol{k}} \Phi_{\Gamma \boldsymbol{k}} c^\dagger_{\boldsymbol{k}} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{R}_j} \qquad (\Gamma = 1,\ 2) \tag{2}$$

§ On sabbatical leave from Rutgers University.

 

that creates an electron at site $j$ in one of two orthogonal Wannier states, with form factor $\Phi_{\Gamma \boldsymbol{k}}$. We are motivated to include a weak second-channel coupling into a Kondo lattice model by the observation that interactions in the conduction generally cause the spin-exchange to spill over from the primary (f-) channel into a weaker, secondary screening channel [10, 11]. We shall also introduce a 'control' model (II), where

$$H_0^{(II)} = \sum_{\boldsymbol{k}\Gamma\sigma} \epsilon_{\boldsymbol{k}} \psi^{\dagger}_{\Gamma \boldsymbol{k}\sigma} \psi_{\Gamma \boldsymbol{k}\sigma} \tag{3}$$

simply describes a band of electrons carrying a conserved channel quantum number $\Gamma = 1, 2$. In the control electrons in different channels do not mix, and the absence of a composite pairing instability in this model provides confirmation that composite pairing effects are a consequence of channel interference.

To examine the effect of second-channel couplings, we introduce the composite operator

$$\Lambda = \sum_j -\mathrm{i}\psi^{\dagger}_{1j} \boldsymbol{\sigma}\sigma_2 \psi^{\dagger}_{2j} \cdot \boldsymbol{S_j} \tag{4}$$

which transfers singlets between channels by simultaneously adding a triplet and flipping the local moment. We now show that channel interference causes the susceptibility of this composite operator to diverge in a Fermi liquid ground-state of channel one.

Suppose $J_2 \ll J_1$ so that a Kondo effect develops in channel one. At low energies the operator $(\boldsymbol{S_j} \cdot \boldsymbol{\sigma}_{\alpha\beta})\psi_{1\beta}$ then behaves as a single bound-state fermion, represented by the contraction

$$\overline{(\boldsymbol{S}_j \cdot \boldsymbol{\sigma}_{\alpha\beta})\psi_{1\beta}}(j) = z f_{j\alpha} \tag{5}$$

where $z$ is the amplitude for bound-state formation. Hybridization between these composite bound-states and conduction electrons forms the heavy-fermion quasiparticles, with energy $E_{\boldsymbol{k}}$ and an enlarged Fermi surface whose enclosed volume counts both conduction and composite f-electrons [12–14].

By applying this contraction procedure we see that the action of the composite operator $\Lambda$ on the heavy-fermion ground-state *creates a pair*:

$$\begin{aligned} \Lambda|\Phi\rangle &= -\mathrm{i} \sum_j \overline{\boldsymbol{S}_j \cdot (\psi^{\dagger}_{1j}}\ \boldsymbol{\sigma}\sigma_2 \psi^{\dagger}_{2j})|\Phi\rangle \\ &= z \sum_{\boldsymbol{k},\sigma} \sigma \psi^{\dagger}_{2\boldsymbol{k}\sigma} f^{\dagger}_{-\boldsymbol{k}-\sigma}|\Phi\rangle. \end{aligned} \tag{6}$$

In the control model, $\psi^{\dagger}_{2\boldsymbol{k}}$ and $f^{\dagger}_{-\boldsymbol{k}}$ are light and heavy electrons on different Fermi surfaces. The mismatch between the decoupled Fermi surfaces for channel one and two ensures that the excitation energy $\epsilon_{\boldsymbol{k}} + E_{\boldsymbol{k}}$ is always finite. By contrast, in the physical model, $\Lambda$ creates a pair of heavy quasiparticles on a *single common* Fermi surface. To see this explicitly we expand both $f_{\boldsymbol{k}}$ and $\psi_{2\boldsymbol{k}} = \Phi_{2\boldsymbol{k}} c_{\boldsymbol{k}}$ in terms of quasiparticle operators $a_{\boldsymbol{k}} = \cos\delta_{\boldsymbol{k}} c_{\boldsymbol{k}} + \sin\delta_{\boldsymbol{k}} f_{\boldsymbol{k}}$. Near the Fermi surface, scattering is resonant, so $\cos\delta_{\boldsymbol{k}_F} \sim 1$, whereas $\sin\delta_{\boldsymbol{k}_F} \propto \Phi_{1\boldsymbol{k}}$ reflects the symmetry of the primary screening channel. Transforming to quasiparticle operators thus introduces a factor $\cos(\delta_{\boldsymbol{k}})\sin(\delta_{\boldsymbol{k}}) \sim \Phi_{1\boldsymbol{k}}$ into the sum, so that near the Fermi surface,

$$\hat{\Lambda} \propto \sum_{\boldsymbol{k},\sigma} \sigma\ \Phi_{1-\boldsymbol{k}} \Phi_{2\boldsymbol{k}} a^{\dagger}_{\boldsymbol{k}\sigma} a^{\dagger}_{-\boldsymbol{k}-\sigma}. \tag{7}$$

This relation describes the decomposition of the composite pair operator in terms of the low-lying quasiparticles. Notice that the operator takes the form of an interference between

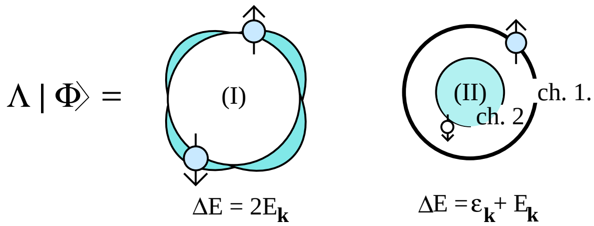


**Figure 1.** Action of composite operator on heavy Fermi liquid creates: (I) a pair of heavy fermions (channel interference) and (II) a heavy and light electron (channel conservation).

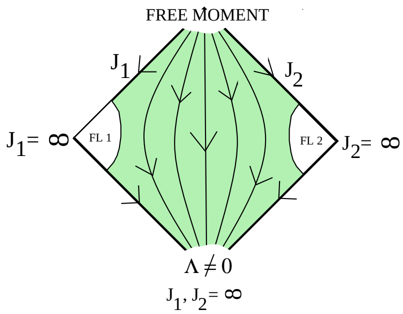


**Figure 2.** Conjectured renormalization group flows for the co-operative two-channel Kondo effect. The Fermi liquid formed in channel one or two is unstable to a common two-channel state with composite order.

the two channels, and furthermore, that the two form factors must have the same parity, or the composite operator vanishes on the Fermi surface. Since the excitation energy, $2E_{\boldsymbol{k}}$ vanishes on the heavy Fermi surface, it follows that there are now a large number of zero modes for the transfer of singlets between channels.

It follows that composite pair susceptibility $\chi_\Lambda$ must contain a singular term, directly proportional to the anisotropic pair susceptibility of the heavy quasiparticles

$$\chi_\Lambda \propto \sum_{\boldsymbol{k}} \tanh\left[\frac{\beta E_{\boldsymbol{k}\lambda}}{2}\right] \frac{(\Phi_{1\boldsymbol{k}}\Phi_{2\boldsymbol{k}})^2}{2E_{\boldsymbol{k}}} \propto \ln\left[\frac{T_{K1}}{T}\right] \tag{8}$$

where $T_{K1}$ is the Kondo temperature for channel one. Any finite $J_2$ will polarize the transfer of singlets into channel two, thereby coupling $J_2$ to this divergent susceptibility. This will cause $J_2$ to scale to strong coupling. A similar conclusion will hold when $J_2$ is large and $J_1$ is small. The simplest way to connect up the renormalization flows in the vicinity of the strong-coupling Fermi liquid fixed points, with the flow away from the weak-coupling fixed point, is by hypothesizing the presence of a new attractive Kondo lattice fixed point that is common to both channels (figure 2).

The main purpose of this paper is to present a simple mean-field realization of this hypothetical two-channel lattice fixed point. We shall show that within our mean-field theory, the BCS-like instability present about the Fermi liquid phase of channel one or channel two leads to a common superconducting phase. One of its distinct features is the development of off-diagonal composite order

$$\langle \hat{\Lambda}(x) \rangle \neq 0. \tag{9}$$

This type of order involves the explicit participation of two screening channels and the local moments in the pair condensate. To explore the nature of this new phase, we present a simple extension of the existing mean-field theory of the Kondo lattice. If we employ the pseudo-fermion representation of the local moments $\boldsymbol{S}_j = \frac{1}{2} f_j^\dagger \boldsymbol{\sigma} f_j$, then the associated constraint $n_f(j) = 1$ at each site leads to a local SU(2) symmetry [14]

$$f_{j\sigma} \to \begin{cases} \mathrm{e}^{\mathrm{i}\theta} f_{j\sigma} \\ \cos\phi f_{j\sigma} + \sigma \sin\phi f_{j-\sigma}^\dagger . \end{cases} \tag{10}$$

The Lagrangian for the f-electrons is

$$\mathcal{L}_f = \sum_j \tilde{f}_j^\dagger \left( \partial_\tau + \boldsymbol{W} \cdot \boldsymbol{\tau} \right) \tilde{f}_j \tag{11}$$

where the tilde field $\tilde{f}_j^\dagger = (f_{j\uparrow}^\dagger,\ f_{j\downarrow})$ denotes a Nambu spinor representation of the f-electron and $\boldsymbol{W}$ is a fluctuating gauge field which imposes the constraint. The SU(2) symmetry makes it possible to simultaneously factorize $H_I$ in both particle–hole and Cooper channels [16]:

$$H_I = \sum_{\Gamma, j} \left\{ \left[ \tilde{f}_j^\dagger \mathcal{V}_j^\Gamma \tilde{\psi}_{\Gamma j} + \mathrm{hc} \right] + \frac{1}{2 J_\Gamma} \mathrm{Tr} [\mathcal{V}_j^{\Gamma\dagger} \mathcal{V}_j^\Gamma] \right\} \tag{12}$$

where $\tilde{\psi}_{\Gamma j}^\dagger = (\psi_{\Gamma j\uparrow}^\dagger, \psi_{\Gamma j\downarrow})$. The field

$$\mathcal{V}_j^\Gamma = \mathrm{i} \begin{bmatrix} V & \Delta \\ -\Delta^* & V^* \end{bmatrix}_j^\Gamma \tag{13}$$

is directly proportional to an SU(2) matrix.

The essential observation is that the onsite product of the two fields $\mathcal{M}(x_j) = \mathcal{V}_j^{1\dagger} \mathcal{V}_j^2$ is invariant under local SU(2) gauge transformations $\mathcal{V}_j^\Gamma \to g_j \mathcal{V}_j^\Gamma$. $\mathcal{M}(x)$ therefore represents a *physical* quantity. Careful re-expression of this matrix in an operator form reveals that its components are directly related to the composite order that develops between the two channels

$$\left\langle \begin{bmatrix} F(x) & \Lambda(x) \\ -\Lambda^\dagger(x) & F^\dagger(x) \end{bmatrix} \right\rangle = \frac{\mathcal{V}^{1\dagger}(x) \mathcal{V}^2(x)}{J_1 J_2} \tag{14}$$

where $F(x_j) = \psi_{1j}^\dagger \boldsymbol{\sigma} \psi_{2j} \cdot \boldsymbol{S_j}$ represents composite charge order and $\Lambda(x)$ is the composite pair density. The product form of this result establishes that composite order is a consequence of interference between the Kondo effect in the two channels.

By removing the site indices on the hybridization and constraint field we obtain the mean-field Hamiltonian

$$H_{MF} = \sum_{\boldsymbol{k}} (\tilde{c}_{\boldsymbol{k}}^\dagger,\ \tilde{f}_{\boldsymbol{k}}^\dagger) \begin{bmatrix} \epsilon_{\boldsymbol{k}} \tau_3 & \mathcal{V}_{\boldsymbol{k}}^\dagger \\ \mathcal{V}_{\boldsymbol{k}} & \boldsymbol{W} \cdot \boldsymbol{\tau} \end{bmatrix} \begin{pmatrix} \tilde{c}_{\boldsymbol{k}} \\ \tilde{f}_{\boldsymbol{k}} \end{pmatrix} \tag{15}$$

where the one-band character of the model forces the order parameter for each channel to enter into the hybridization $\mathcal{V}_{\boldsymbol{k}} = \mathcal{V}^1 \Phi_{1\boldsymbol{k}} + \mathcal{V}^2 \Phi_{2\boldsymbol{k}}$. This provides the origin of the interference between the two channels. Choosing the gauge where $\mathcal{V}^1 = \mathrm{i} v_1 \underline{1}$, then a stable composite-paired solution emerges with $\mathcal{V}^2 = v_2 \underline{\tau_1}$ and $\boldsymbol{W} = (0, 0, \lambda)$. After some work, we find that the eigenvalue spectrum of (15) has two branches, where

$$E_{\boldsymbol{k}\pm} = \sqrt{\alpha_{\boldsymbol{k}} \pm (\alpha_{\boldsymbol{k}}^2 - \gamma_{\boldsymbol{k}}^2)^{\frac{1}{2}}} \tag{16}$$

where $\alpha_{\boldsymbol{k}} = V_{\boldsymbol{k}+}^2 + \frac{1}{2}(\lambda^2 + \epsilon_{\boldsymbol{k}}^2)$, $\gamma_{\boldsymbol{k}}^2 = [\lambda\epsilon_{\boldsymbol{k}} - V_{\boldsymbol{k}-}^2]^2 + (2v_{1\boldsymbol{k}}v_{2\boldsymbol{k}})^2$ and we have defined $v_{\Gamma\boldsymbol{k}} = v_\Gamma\Phi_{\Gamma\boldsymbol{k}}$, $V_{\boldsymbol{k}\pm}^2 = v_{1\boldsymbol{k}}^2 \pm v_{2\boldsymbol{k}}^2$. The requirement that the free energy per site

$$F = -2T \sum_{\boldsymbol{k},\alpha=\pm} \ln\left[2\cosh(\beta E_{\boldsymbol{k}\alpha}/2)\right] + \sum_{\Gamma=1,2} \frac{(v_\Gamma)^2}{J_\Gamma} \tag{17}$$

is stationary with respect to variations in $v_2$, $v_1$ and $\lambda$ gives rise to three mean-field equations. Two classes of solution exist:

- *Normal state*: $v_1$ or $v_2 = 0$. Two normal state phases exist corresponding to a single-channel Kondo effect in channel one or two. The Fermi surface geometries of the two phases are topologically distinct, and at half filling these phases evolve into two different Kondo insulating phases.
- *Composite paired state*: $v_1v_2 > 0$. When channel conservation is absent, a Kondo effect in both channels leads to a paired state with an anisotropic heavy-electron gap function $\Delta_{\boldsymbol{k}} \sim \sqrt{T_{K1}T_{K2}}\Phi_{\boldsymbol{k}1}\Phi_{\boldsymbol{k}2}$.

Setting $v_2 = 0^+$ in the mean-field equations, the transition from the one-channel Fermi liquid into the composite paired state is given by $J_2\chi_\Lambda(T_c) = 1$ where

$$\chi_\Lambda(T) = \sum_{\boldsymbol{k}\alpha} \tanh\left(\frac{E_{\boldsymbol{k}\alpha}}{2T}\right)\frac{(\Phi_{\boldsymbol{k}2})^2}{2E_{\boldsymbol{k}\alpha}}\left[1 + \frac{(\lambda - \epsilon_{\boldsymbol{k}})^2}{(E_{\boldsymbol{k}\alpha}^2 - E_{\boldsymbol{k}-\alpha}^2)}\right] \tag{18}$$

is the composite pair susceptibility. There are two important contributions to this integral: a high energy, single-ion part where $E_{\boldsymbol{k}+} \sim |\epsilon_{\boldsymbol{k}}| \gg T_{K1}$ and a low energy 'Fermi surface' contribution where the term in square brackets is proportional to $(\Phi_{1\boldsymbol{k}})^2$, so that

$$\chi_\Lambda \approx 2N(0)\left[\langle\Phi_{2\boldsymbol{k}}^2\rangle \ln\left(\frac{D}{T_{K1}}\right) + \langle\Phi_{1\boldsymbol{k}}^2\Phi_{2\boldsymbol{k}}^2\rangle \ln\left(\frac{T_{K1}}{T}\right)\right] \tag{19}$$

where $\langle\ldots\rangle$ denotes an angular average, $D$ and $N(0)$ are the conduction electron bandwidth and density of states respectively. Notice how the second interference term largely compensates for the single-ion cut-off ($T_{K1}$) in the first term. A composite pair instability occurs at

$$T_c \sim D(D/T_{K1})^{\zeta-1} \exp\left[-\frac{1}{2\langle\Phi_{1\boldsymbol{k}}^2\Phi_{2\boldsymbol{k}}^2\rangle N(0)J_2}\right] \tag{20}$$

where $\zeta = \langle\Phi_{2\boldsymbol{k}}^2\rangle/\langle(\Phi_{1\boldsymbol{k}}\Phi_{2\boldsymbol{k}})^2\rangle$.

To illustrate this conclusion we have used a two-dimensional model where the local moments couple to a tight-binding lattice of conduction electrons via an 's' and 'd' channel:

$$\Phi_{1\boldsymbol{k}} = 1 \qquad \Phi_{2\boldsymbol{k}} = [\cos(k_x) - \cos(k_y)]. \tag{21}$$

Figure 3 shows the phase diagram computed using the mean-field equations. When $J_2 \sim J_1$ the mean-field transition temperature for composite order is comparable with the single-site Kondo temperature.

An interesting prediction of the theory is the existence of a second-order superconducting–insulating transition. At half-filling the normal state is a Kondo insulating ground-state in channel one or two. Beyond a critical value $J_2 > J_2^*$, a Kondo insulator in channel one becomes unstable with respect to a composite-paired state. Even though this phase forms in the complete absence of a Fermi surface, the superfluid stiffness

$$\rho_s = \left(\frac{2}{d}\right)\sum_{\boldsymbol{k}}\left(\frac{v_1v_2}{V_{\boldsymbol{k}+}}\right)^2 \frac{(\Phi_{1\boldsymbol{k}}\nabla\Phi_{2\boldsymbol{k}} - \Phi_{2\boldsymbol{k}}\nabla\Phi_{1\boldsymbol{k}})^2}{[(\epsilon_{\boldsymbol{k}}/2)^2 + V_{\boldsymbol{k}+}^2]^{\frac{1}{2}}} \tag{22}$$

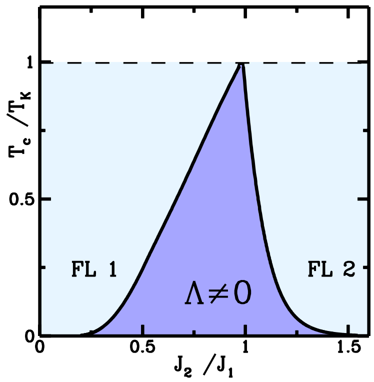


**Figure 3.** Phase diagram for a two-channel Kondo lattice with 's' and 'd wave' screening channels. Composite pairing develops in shaded region.

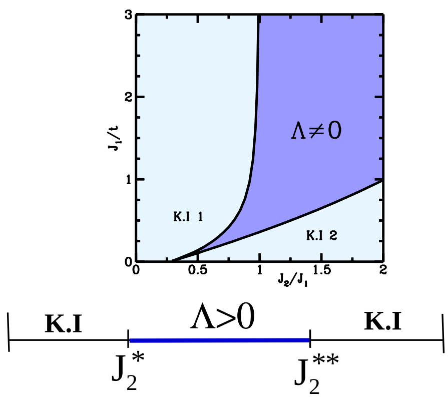


**Figure 4.** Phase diagram for a two-channel Kondo insulator. 'K.I 1' and 'K.I 2' denote Kondo insulating phases in channel one and two respectively. In the intermediate gapless phase both channels participate coherently in the composite pairing process.

is positive (where $d$ is the dimensionality). At a higher value $J_2 > J_2^{**}$, the Kondo effect in channel one is finally suppressed, forming a second Kondo insulating state. Figure 4 shows how the Kondo-insulating ground-states become unstable to a composite paired state at strong coupling.

In closing, it is perhaps instructive to contrast composite and magnetically mediated pairing [17, 18]. The latter is maximized in the vicinity of an anti-ferromagnetic quantum-critical point. By contrast, the composite pairing described here is driven by a constructive interference between two rival normal phases, and requires no fine tuning. The gap function is determined by an interference product of two Wannier functions, $\Delta_{\boldsymbol{k}} \propto \Phi_{\mathbf{1}\boldsymbol{k}}\Phi_{\mathbf{2}\boldsymbol{k}}$, predicting an intimate relationship between the gap symmetry and local quantum chemistry. When the primary spin exchange occurs in the f-channel, a small exchange coupling to a p-channel will develop a composite paired state with a gap symmetry $\Phi_{\mathrm{f}} \times \Phi_{\mathrm{p}}$. For transition metal systems, admixture between a primary d-channel and a secondary s-channel will provide a gap with d-symmetry.

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